Undergraduate Research Opportunity Programme in Science

Lunar Visibility and the Islamic Calendar

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Contents

1 Introduction

1.1 Basic Calendrical Concepts

Calendars are used to keep track of the motion of the earth, moon and sun. They are formed from the units of the day, month and year. The day arises due to the rotation of the earth about its axis; the month arises due to the revolution of the moon around the earth; while the year arises due to the revolution of the earth around the sun respectively. We shall discuss some basic calendrical concepts below.

The Earth and the Seasonal Markers

The Earth revolves anticlockwise around the sun in an elliptical orbit in an ecliptic plane, known as the plane of ecliptic. The ecliptic is the path of the earth around the sun on the celestial sphere. It makes an angle of 23.5° with the celestial equator. Simultaneously, the earth rotates anticlockwise around an axis that is tilted 23.5° with respect to the line perpendicular to the ecliptic. The tilt of the axis and the revolution of the earth around the sun give rise to seasons.

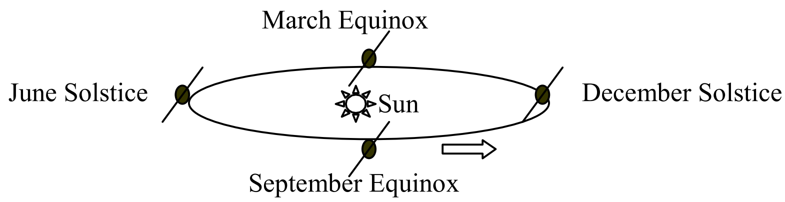


Fig. 1

The ecliptic intersects with the celestial equator at two points, known as the equinoxes. The solstices, on the other hand, are where the equator and the ecliptic are furthest apart. The equinoxes and the solstices are known as the seasonal markers. Let us look at the overall picture of the celestial sphere.

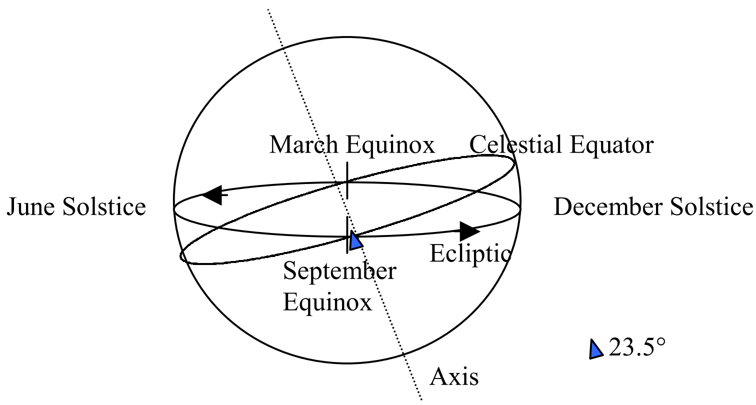
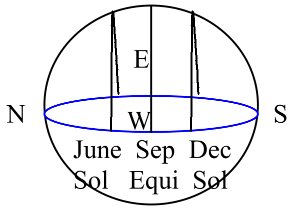


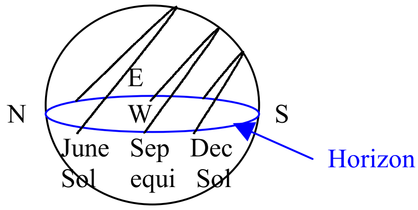
Fig. 2

March equinox (March 21) is the time when the sun passes the celestial equator from the southern to the northern hemisphere. Here, the length of day and night is equal. After the March equinox, the sun moves north until the June solstice

(June 22), where the northernmost rising position is. Here, people staying in the northern hemisphere experience the longest day, while those staying near the equator experience the same length of day and night because the sun rises from east to west for about the same length of time. Figure 3 shows the path of the sun in the sky for observers in Beijing and Singapore.



Near the Equator
(eg Singapore)
Fig. 3a



Higher latitude region
(eg Beijing)
Fig. 3b

The sun then continues to move southward. As it reaches the September equinox (September 23), the day and night are almost equal in length again. The days shorten as the sun moves south until it reached the December solstice (December 22) where the southernmost rising position is. At this point, people staying in the northern hemisphere experience the shortest day.

Perihelion (near December solstice) is the point when the earth is closest to the sun. It occurs around 4^{th} January every year. Aphelion (near June solstice) is the point when the earth is furthest away from the sun and occurs around 4^{th} July.

The Coordinate Systems

Let's look at the three kinds of coordinate systems.

a) <u>The Equatorial Coordinate System</u>

Here, we measure the star's altitude with respect to the celestial equator, from 0° at the equator to 90° at the North Pole, and to -90° at the South Pole. This is called the **declination**. The declination corresponds to the terrestrial latitude.

The other positional coordinate of this system is known as the **right ascension** (RA). The right ascension is the arc distance (in degrees) measured from the point of March equinox (0°) along the celestial equator in the right-hand-side direction to the projection of the star onto the celestial equator. The right ascension corresponds to the terrestrial longitude.

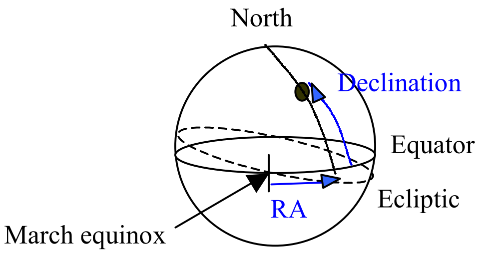


Fig. 4

b) The Ecliptic Coordinate System

Here, the star's positional coordinate is measured with respect to the ecliptic. **Celestial latitude** is measured perpendicularly from the ecliptic (0°) to the north (90°) and south (90°) ecliptic points, while **celestial longitude** is measured from the point of March equinox (0°) in the same way as the right ascension, except that it is along the ecliptic.

c) The Horizon Coordinate System

Altitude (Z) is the arc distance measured from the star to the horizon, while **azimuth (Az)** refers to the arc distance (in degree) measured from the north along the horizon to the projection of the star onto the horizon.

Zenith

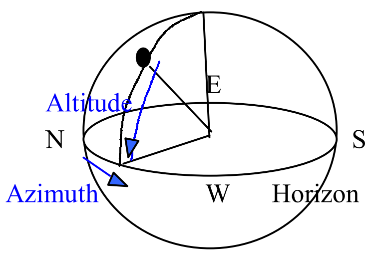


Fig. 5

Basic Quantity

We will see in the later part of the discussion that the relative positions of the sun and the moon are important in determining the visibility of the crescent moon. Let us define some basic quantity.

Relative azimuth (ΔAz)
This refers to the difference in azimuth between the moon and the sun

Moon's altitude at local sunset (ΔZ or h)
This refers to the height of the moon above the local horizon.

Solar depression (s)
This refers to how low the sun is below the horizon.

Moonset lag
Moonset lag is the time interval between sunset and moonset

Arc of light (a_L)
The arc of light is defined as the elongation or angular distance between the sun and the moon.

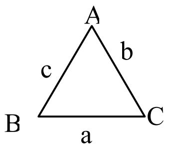
Arc of separation (a_S)
The arc of separation is the difference in RA (equatorial separation) between the sun and the moon, measured in degrees.

Spherical Trigonometry

The spherical trigonometric formulae are used to solve astronomical problems relating to the celestial sphere. Here, we will talk about the cosine formula to understand some basic concepts.

***The cosine formula*

cos a = cos b cos c + sin b sin c cos A *(Spherical Trigonometry)*



We will see how the arc of light is related to the cosine formula below:

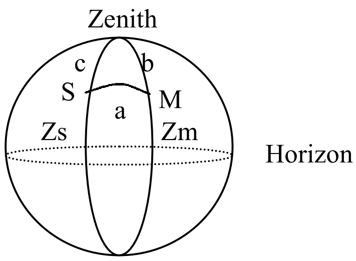


Fig. 6

The above is the celestial sphere, where $a = a_L$, $b = (90^\circ - Z_m)$, $c = (90^\circ - Z_s)$,
Applying the cosine formula,

$$\mathbf{\cos a_L} = \cos(90^\circ - Z_m)\cos(90^\circ - Z_s) + \sin(90^\circ - Z_m)\sin(90^\circ - Z_s)\cos(\Delta Az)$$
$$= \mathbf{\sin Z_s \sin Z_m + \cos Z_s \cos Z_m \cos(\Delta Az)}$$

(Z_s = altitude of sun, Z_m = altitude of moon)

A slight variation to the above shows the relationship between the arc of light and the relative azimuth.

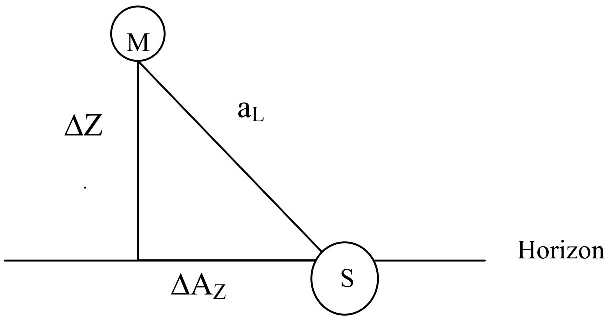


Fig. 7

$$\mathbf{\cos a_L = \cos \Delta Z \cos \Delta A_Z + \sin \Delta Z \sin \Delta A_Z \cos 90^\circ}$$
$$= \cos \Delta Z \cos \Delta A_Z + 0$$
$$\Rightarrow \quad \Delta A_Z = \cos^{-1}(\cos a_L / \cos \Delta Z)$$

In addition to the above cosine formulae, the relationship between the arc of light and the width of the crescent is also an important concept.

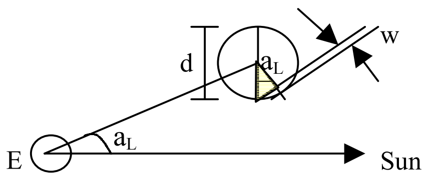


Fig. 8

$$\mathbf{w} = d/2 - d/2\,(\cos a_L)$$
$$= d/2\,(1 - \cos a_L)$$
$$\mathbf{= d\ sin^2\,(a_L\,/2),\ d = lunar\ diameter\ in\ km}$$

The Moon, Conjunction and Crescent Visibility

The moon can be seen because it reflects sunlight. It revolves anticlockwise around the earth and this allows us to see the different phases of the moon. At *conjunction*, where the difference between the longitude of the sun and longitude of moon is 0°, the moon sets at almost the same time as the sunset. We call this moon the new moon. A few evenings after conjunction, the crescent will be visible. Gradually, as the moon revolves around its orbit around the earth, the moon will set after sunset. There comes a point when the moon falls in the same line as the sun and earth but on the opposite side of the earth. This moon is called the full moon. At this time, the moon sets at sunrise and rises at sunset. The moon continues to revolve around the earth until the new moon occurs again, and the cycle repeats. The time period between one new moon and the next new moon is known as *lunation*, and varies between 29.27 and 29.84 days.

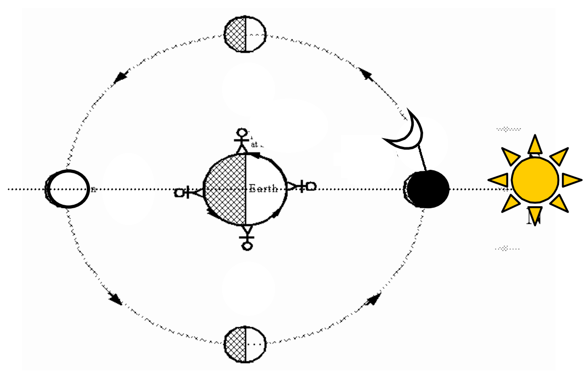


Fig. 9

Conjunction can occur when the moon is directly in front of the sun at new moon or 5° away, as long as their celestial longitude of the moon and sun is the same. At higher latitudes (see fig. 3), the sun sets at a smaller angle to the horizon compared to that near the equator. Since the moon revolves near the eclliptic (0° to 5° difference), it also behaves similarly to the sun and sets at a smaller angle to the horizon. Near the equator, the sun sets almost perpendicularly to the horizon. The moon, too, sets almost perpendicularly to the horizon.

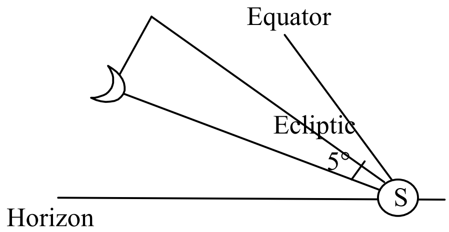


Fig. 10

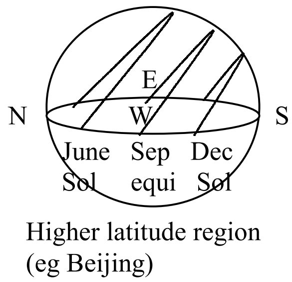


Fig. 3b

The Tropical Year

In the past, tropical year was defined as the time interval between two consecutive March equinoxes. However, it was found that the time interval between two March equinoxes, two June solstices, two September equinoxes and two December solstices are slightly different. (For more details, refer to http://www.math.nus.edu.sg/aslaksen/calendar/) Hence, to define it more accurately, the tropical year is the time interval for the sun's mean longitude to increase by 360°. Its length is about 365.2422 days.

1.2 Solar, Lunar and Lunisolar Calendars

Solar Calendars

Solar calendars aims to ensure that the seasonal markers do not move in the calendar year by adding leap days. They ignore the movement of the moon and only follow the movement of the sun. One example of such calendars is the Gregorian calendar, which is the most commonly used calendar today. The goal of the Gregorian calendar is to make the average Gregorian year equal the tropical year, so that the seasonal markers stay almost constant. However, a normal Gregorian year consists of 365 days and the tropical year consists of 365.2422 days. Since each normal year is about a quarter day shorter, the equinox moves forward a quarter day for three consecutive years. A leap year rule is then introduced to even it out at the fourth year. A leap year occurs when a particular year is divisible by 4 and not 100 or divisible by 400, and in a leap year, an extra day is added to February. For instance, 2000 is a leap year, while 1900 is not. One Gregorian year is divided into 12 months, with each month having a fixed number of days. The basic unit of the solar calendars is days.

Lunar Calendars

Lunar calendars aims to ensure that conjunction, crescent visibility, or full moon determines the start of the month. The basic unit of lunar calendars is the lunar month. Lunar calendars follow the lunar phase cycle and ignore the movement of the sun, hence, lunar years do not follow the seasons. The only lunar calendar used today is the Islamic calendar. A lunar year of the Islamic calendar consists of 12 months, with each month beginning with the first sighting of the new moon. (Muslim

calendars are based on crescent visibility and not conjunction for religious reasons.) Since the Islamic calendar is not related to the tropical year, Islamic dates move back about 11 days each year. This is because each Muslim month consists of 29.5 days on average. Hence, in a Gregorian year, there are about 12 x 29.5 = 354 days, i.e. the 12 lunar months per year “slips” away by about 11 days with respect to the solar year.

Lunisolar Calendars

Lunisolar calendars aims to approximate the tropical year by adding leap months. One example of such is the Chinese calendar, which consists of 12 months that begin at the new moon. A 13^{th} month (leap month) is added a little less than every 3 years so that the calendar stays in line with the seasons. The basic unit of lunisolar calendars is the lunar month.

2 The Islamic Calendar and its History

2.1 The Islamic Calendar

Unlike the Gregorian calendar, which serves as an international standard for civil use, there are other calendars which are used for religious and cultural purposes, and are interesting for further study. One of them is the Islamic Calendar, which is strictly a lunar calendar. For religious reasons, the Quran specifies that the Muslim months start with the sighting of the new moon at a given locale.
Each year consists of 12 lunar months with no leap months and the names of the lunar months are:

1. Muharram
2. Sarfar
3. Rabiʻa I
4. Rabiʻa II
5. Jumada I
6. Jumada II
7. Rajab
8. Shaʻban
9. Ramadan
10. Shawwal
11. Dhu al-Qʻadah
12. Dhu al-Hijjah

For the Islamic calendar, there are 7 days in a week, with each day beginning at sunset. Day 1 begins at sunset on Saturday and ending at sunset on Sunday. Day 5, which is calle Jumʻa, is a day for congregational prayers.

Determining the first day of every Islamic month is very crucial as it determines when the Muslim festivities will fall on. For instance, Muslims need to know when is the first day for fasting in the Ramadan month and when fasting ends (i.e. when to celebrate Hari Raya Puasa). Hence, many practices have been adopted to determine the first day of each lunar month.

Since the Islamic calendar is not related to the tropical year and has no fixed relationship with the seasons as mentioned earlier, important Muslim festivals which always fall in the same Muslim month, occur in different seasons. For example, Hari Raya Puasa in 1998 fell on 30/1/1998; in this year, it falls on 16/12, and will continue

to fall back 11 days earlier every year until it takes place in a different season. It is only over a 33-year cycle that Hari Raya Puasa will fall on almost the same day as the first year again. This is because 33x11 ≈ 365 days.

2.2 The History of Islamic Calendar

The history of the Islamic calendar dated back to as far as the seventh century AD, when Prophet Mohammed announced the use of a lunar Islamic calendar. But before the days of Prophet Mohammed (the **pre-Islamic era**: AD 570 – 632), the Arabs used a **lunisolar** calendar. This was because they found that the lunar calendar does not keep step with the seasons and thus, caused some difficulties. When they made a pilgrimage to Mecca in the 12th lunar month, they had to bring along an animal for slaughter as an act of sacrifice. However, with the lunar calendar not being able to keep up with the seasons, the Arabs had difficulties in finding food (crops could not be harvested) for the pilgrimage trip as well as the animals for slaughter. As such, they inserted a 13th month occasionally in order to keep the 12th in the autumn. As to when to insert the 13th month, it was decided by the calendar officials (the Nasa'a) of the Kinana tribe, but exactly when it was inserted was not known.

Prophet Mohammed announced the strict use of a **lunar** calendar shortly before his death in AD 632. It was only in AD 642 when Caliph Umar I set up the calendar and took 16 July AD 622 as the starting date of the **Islamic era**. This era is known as the era of *Hijra* (AH[1]), where 622 was chosen to commemorate the *Hijra*, the date of Prophet's move from Mecca to Medina. (AD 622 = AH 1). However, this date was not the exact arrival date, though the two dates are probably near to each other.

3 Hari Raya Puasa and Hari Raya Haji

The two Islamic holidays in Singapore are Hari Raya Puasa and Hari Raya Haji. Hari Raya Puasa falls on the 1st day of the 10th Islamic month while Hari Raya Haji falls on the 10th day of the 12th Islamic month.

Since an Islamic year is shorter than a Gregorian year, the time period between any same Muslim dates is less than a Gregorian year. Therefore, any Muslim festivals can occur twice in the same Gregorian year. For instance, in 1999, Hari Raya Puasa fell on Jan 19; in 2000, it fell on Jan 8 and Dec 27. Since 365/11 ≈ 33, this "twice" Hari Raya Puasa per year will occur every 32 or 33 years. Also, Hari Raya Puasa will meet up with Chinese New Year for 2-3 years, and then continue on its 33 years cycle backwards through the calendar and meet up with Chinese New Year again after about 30-31 years.

4 Different Approaches to the Islamic Calendar

[1] AH is Anno Hegirae in full and means the year of Hijra.

4.1 The Arithmetical Islamic Calendar

Arithmetical Islamic calendars are constructed based on arithmetical calculations to make rough predictions of lunar visibility, where the number of days in a month alternate between 29 and 30. Each lunar year consists of 354 days (355 in leap years). In a leap year, an extra day is added to the 12th Islamic month to keep the calendar in step with the true phases of the moon.

Leap years are years that follow the following algorithm: the number *year mod 30* is one of the following: 2, 5, 7, 10, 13, 16, 18, 21, 24, 26, or 29. The table below shows a cycle of 30 lunar years from year 1 to 30. The total number of days in this system of 30 Islamic years = (354x30 + 11) = 10631; dividing this figure by 360 (30 years of 12 months), we will get the average lunation of the moon to be 29.53 days.

Year	No of Days	Year	No of Days	Year	No of Days
1	354	11	354	21	**355**
2	**355**	12	354	22	354
3	354	13	**355**	23	354
4	354	14	354	24	**355**
5	**355**	15	354	25	354
6	354	16	**355**	26	**355**
7	**355**	17	354	27	354
8	354	18	**355**	28	354
9	354	19	354	29	**355**
10	**355**	20	354	30	354

Now, when we are considering of constructing an Islamic calendar, we need to consider two conditions. Firstly, the mean lunation (= 29.53 days) is not whole number, yet, we need to assign an integral number of days to each month of the calendar. In this aspect, the arithmetical calendar has made a very good approximation as calculated above. However, the second thing that we have to consider is the variation of the lunation. This variation ranges between 29.27 and 29.84 days. That means if we adopt this arithmetical system, there is a possibility that the new moon will occur one day before or after the proposed alternate dates. In this aspect, the arithmetical Islamic calendar is totally unacceptable. And it's because of this that none of the Muslim community uses this fixed system.

Kepler's second law states that the radial line sweeps out equal areas ($A_1 = A_2$) in equal time ($t_1 = t_2$); and since the earth is nearest to the sun at perihelion (near winter solstice), it follows that $d_1 > d_2$. As a result, the earth moves at a greater speed ($s_1 > s_2$). As the moon revolves around the earth, the moon has to take a longer time to "catch up" with the faster motion of the earth around the sun, i.e. the lunation will be longer.

In contrast, at aphelion (near summer solstice), since the earth is furthest away from the sun where there is less gravitational pull, the earth revolves around the sun at a slower speed. While the moon is moving around the earth, it requires a shorter time period to "catch up" with the motion of the earth, i.e. lunation is shorter.

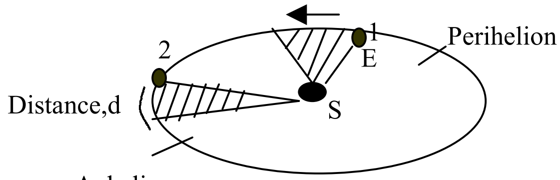


Fig. 11

Therefore, the time the new moon occurs depends on where the period of the year and cannot be based on a fixed number of days every month.

4.2 Islamic Calendar based on Observations

Traditionally, the first day of each Islamic month is determined based on the first sighting of the new moon. Sighting of the new moon is of religious importance among Muslims. However, there are many factors that affect the visibility:

Astronomical factors include:

1. Large moonset lag
2. Moon too young
3. Low altitude of the moon at sunset
4. Crescent width / elongation of moon from sun
5. Latitude and longitude of observer

Non-astronomical factors include:

1. Poor weather conditions
2. Atmospheric pollution
3. Poor eyesight of the observer / Use of optical aids

Due to all these, festive celebrations are sometimes held on different days by different countries, or even by different communities staying in different areas of the country.

4.3 Islamic Calendar based on Predictions

This calendar is based on astronomical data and theory. With the hope that Islamic festivals can be celebrated together on the same day within the whole country or community, predictions of when the crescent will be seen have been made. Nevertheless, it is not easy to predict the calendar or convert from one kind to another. Different criteria are being developed over the years to determine the beginning of the lunar months with greater accuracy and will be discussed later.

5 Practices in Different Countries

Traditionally, the first visibility of the new moon to the naked eye marks the beginning of the month.

5.1 Physical Sightings

Some countries use sighting claims and have judges (Qadi) in different parts of the country to accept or reject claims. Example: India, Pakistan, Bangladesh, and Morocco.

Some countries use the calculations with the convention that if the moonset occurs after sunset (conjunction occurs), then the next day is the first day of the Islamic Month. Example: Saudi Arabia, which generally uses this method but sometimes deviates from this for the months of Ramadan, Shawwal, and Zul-Qa'dah, by using claims of sighting, or other reasons.

5.2 Calculations / Astronomical Criteria

Some countries use purely calculations with the convention that if the new moon birth has occured and the moon sets 5 minutes after Sunset, then the next day is the first day of the month. Example: Egypt, where calculations are done for the Western boundaries of the country, which have a better chance of seeing the moon than other parts of the country.

Some countries adopt Mabims criteria by calculations, such as moon's age ≥ 8 hours, altitude above horizon ≥ 2 degrees, elongation ≥ 3 degrees at Sunset, then the following day is the first day of the Islamic Month. Examples: Malaysia, Singapore, Brunei and Indonesia.

Some countries adopt calculations with specific criteria, such as moon's age, altitude above horizon, interval between sunset and moonset. If the moon fulfills these criteria at sunset, then the following day is the first day of the Islamic Month. Examples: Iran, Algeria, and Tunisia.

5.3 Physical Sightings and Calculations

Some countries adopt the actual sighting of the crescent with the assistance of astronomical calculations to screen out the mistaken claims and erroneous sightings. This is the most reliable and accepted method among scientists, as well as among most of jurists (Fuqahaa') of Islamic Shari'ah in recent times, and Jordanian Astronomical Society adopted this method. Recently, many organizations in Trinidad & Tobago, and Guyana have also adopted this method.

5.4 Following Other Countries

Some countries follow the news of the beginning of the month from Saudi Arabia. Examples: Qatar, Kuwait, UAE, Oman, Bahrain, Yemen, Turkey, Libya, and Afghanistan.

Some small countries follow the news of the beginning of the month from larger neighboring countries. Examples: New Zealand following the news from Australia, and Suriname following the news from Guyana.

Some countries follow the news of the beginning of the month from the first Muslim country that announces it. Examples: many Europian countries, and some Carribean Islands.

Some countries/faith groups adopt pre-calculated calendar. Example: Bohra community, Ismaiili community, and Qadiani community – the Muslim communities located at different parts of the world like Pakistan and India.

5.5 No Specific Method

Some countries don't adopt any specific method. Their decision varies year by year according to different methods. Example: Nigeria.

6 Prediction Criteria - Their Background and Reliability

Although it is possible to calculate the *position* of the moon in the sky with high precision, it is often difficult to predict if a crescent will be *visible* from a particular location due to the factors affecting visibility.

Crescent visibility is a tough problem, involving orbital calculations, lunar scattering, atmospheric scattering and visual physiology. Since the Babylonians era, as far back as 500 C.E., astronomers have been trying to find ways to predict when the new moon can be first seen every month. A simple criterion was developed and had been in use for centuries without much improvement. It was only until 1910, when the astronomer, J.K. Fotheringham deduced another set of criteria, which forms a crucial part of the development in lunar visibility prediction. While Fotheringham made use of fully observational data obtained from Athens, in 1977, another astronomer, Frans Bruin improvised on it with more theoretical basis. In 1984, Mohammad Ilyas, a research astronomer, then made further improvement to his criterion.

In the late 1980s, data was collected systematically from across North America. 1784 observations were collected and tested against various prediction criteria.

On 26 May 1990, a record breaking of a 15.0 hr young moon (thin crescent) was sighted with naked eye; and on 21 January 1996, the young moon of 12.1 hr as seen through an 8-inch telescope!

In this section, the computer programme (MoonCalc), developed by Dr Monzur Ahmed for predicting lunar visibility based on the different criteria, will be tried out and some illustrations will also be shown. Let us discuss the various criteria in more details.

6.1 Babylonian (Age at sunset >24hrs, Moonset Lag >48 min) 500 C.E.

This is the earliest astronomical criterion for ascertaining the crescent's first visibility. In ancient times, the Babylonians developed a set of moon sighting criterion where the moon will be visible when

(i) At local sunset, the age of the moon is >24 hours, from the time of conjunction to the time of observation,

(ii) and that the sunset to moonset interval is > 48 minutes (a_s>12° at sunset)

Why is it 12°?
Suppose the sun and the moon set together, since we require the moon to be >24 hrs old in order to be seen and the fact that the moon take a month (≈30 days) to make a complete revolution (360°) around the earth, hence,
In 30 days – 360°
In 24 hrs – 12°
The moon lags 12° behind the sun.

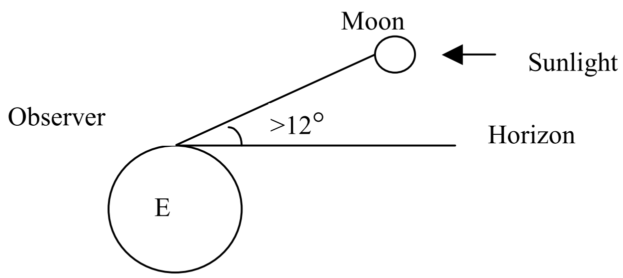


Fig. 12

Why does 12° correspond to 48 minutes?
This is because the earth makes a full rotation of 360° in a day (24 hr).
Hence,
24hr – 360°
1 hr - 15°
48min – (48/60) x 15° =12°

Moreover, from the prediction testing as mentioned above, (Doggett and Schaefer, 1994) a histogram of fraction wrong vs moon age is drawn, and it turned out that the model was extremely biased, with more than 50% error for moon age between 18 and 24 hrs old as well as frequent incorrect predictions over the whole range, showing poor accuracy.

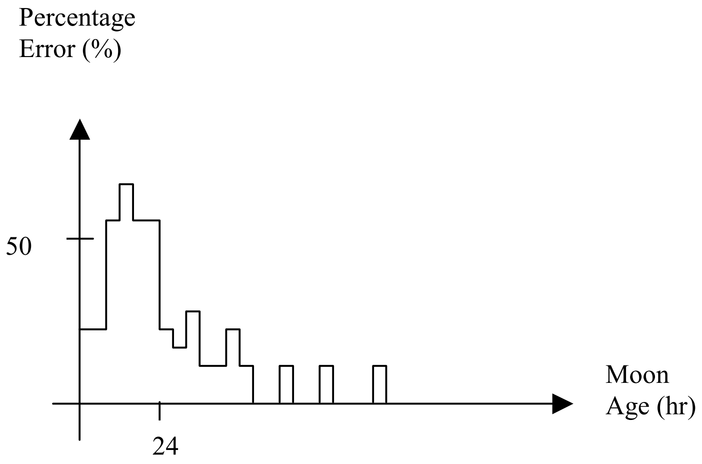


Fig. 13

The record breaking of a 15.0 hr (<24 hr) young moon seen with naked eye and the fact that a 51.3 hr old moon had been missed further show the inaccuracy of the simple moon age criterion.

As for the moonset lag (>48 min) criterion, a histogram of fraction wrong vs moon age is drawn, and it turned out that there is a >50% error for moonset lags between 50-55 min, which is also a biased model.

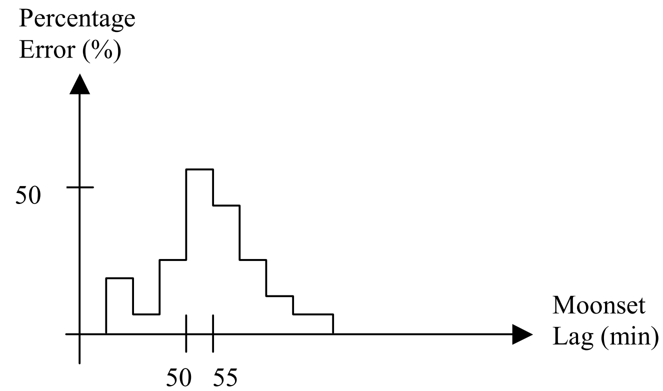


Fig. 14

In addition, this criterion is a simple observational criterion that does not take into account the position of the observer. At higher latitudes (see fig. 3b and fig. 10), the sun sets at a smaller angle to the horizon compared to that at near the equator. As a result, the moonset lag is much shorter. Before the sun has fully set (i.e. while the sky is still bright), the moon is already setting, and hence, the moon cannot be seen. However, near the equator, the sun sets almost perpendicular to the horizon. In the case, the sun sets very quickly and the sky gets dark much faster way before the moon sets. Therefore, the moon can be seen. The 48 min moonset lag has not taken this latitude condition into account and is not an accurate measurement. Moonset lags as short as 35 min have been seen and lags as large as 75 min have been invisible.

Therefore, the Babylonians' criterion of moon age and moonset lag is unreliable.

6.2 Fotheringham (Moon's Altitude, Relative Azimuth between moon and sun) 1910

In 1910, Fotheringham made a collection of 76 naked eye observations of the visibility and non-visibility of the crescent from 1859 – 1880, using mainly J. Schmidt's log at Athens. Schmidt was an astronomer in Athens and had contributed to the development of the Islamic calendar through his many observational data (see Schmidt, J., 1868, *Astr. Nachr*, On the earliest visibility of the lunar crescent in the evening sky). For each observed data set, Fotheringham calculated the moon's altitude and the relative azimuth separation at local sunset, and plotted a curve. The curve showed a clear dividing line between the positive (points above the curve) and the negative (points below the curve) observations.

To apply this criterion, a set of data of moon's altitude and relative azimuth is collected. If this set of data lies above the curve, then the crescent is visible. If the data lies below the curve, then the crescent is likely to be not visible.

From the curve, we observe that as the relative azimuth increases, the required moon altitude for it to be seen decreases, but the moon's altitude is required to be ≥ 6°. That is, the moon will not be visible if it is just above the local horizon. However, this criterion assumes that every part of the world had the same clarity of air as in Athens, be it the humid lowlands of East India or the high mountains of North India. It does not account for other important factor like seasons, latitudes, elevation, humidity and atmospheric clarity. And because these factors change with time of the year and location drastically, the threshold value of 6° not only varies from site to site, but also from month to month (lunation to lunation).

In addition, the calibrating of the dividing line based on all the observations might also cause inaccuracies. For moon directly over the sun, the greatest value for the moon's altitude is approximately 12° for Fotheringham, 11° for Maunder, 10.5° for Ilyas A and C (to be discussed later).

6.3 Maunder (Moon's Altitude, Relative Azimuth between moon and sun) 1911

In 1911, E.W. Maunder, a pioneering British astronomer, used Schmidt's data, together with more observations and plotted a curve lower than Fotheringham's. This is because Schmidt's observations at Athens were usually confined to older crescents, hence, positive observations were more liable to be mistaken than negative observations.

Similar to Fotheringham, the data were more accurate up to low latitudes. At higher latitudes where the atmosphere is not as clear, modification is needed.

6.4 Indian (Moon's Altitude, Relative Azimuth between moon and sun) 1900 C.E.

The Indian scientists from the Indian Astronomical Ephemeris plotted a curve which is slightly lower than Maunder's using a slightly modified criterion. (see Ashbrook 1971, Sky & Telescope)

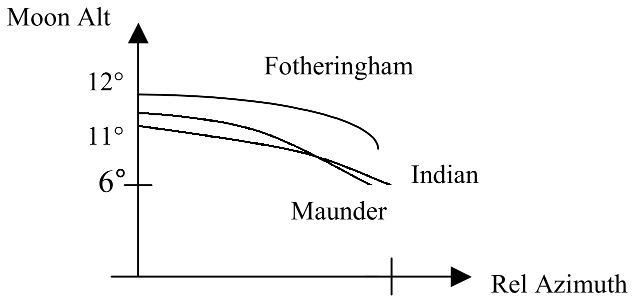


Fig. 15

6.5 Bruin (Moon's Altitude, Crescent Width) 1977

In 1977, Bruin developed a set of criterion which was based mainly on theory. This criterion of his is based on the crescent width and the moon's altitude at sunset, while taking into account illumination of sky, light intensity of crescent, crescent width, solar depression, contrast for human eye, humidity, elevation, etc. The width of the moon is suggested to be >0.5 arcmin for possible visibility.

Data was collected and a graph of h (moon's altitude), h+s is plotted against s (solar depression) for chosen values of w (or a_L, elongation). To determine visibility, find the correspondent h+s data for a given width. If the curve (data) lies above the curve, then there is visibility. For instance, if we find that on a certain evening after a new moon, h+s = 10°, we draw a horizontal line across the curves. This line will intersect at 2 points, A and B. The crescent will be visible for the first time at A and will remain visible for 6° of solar dip (about 30 minutes) until point B.

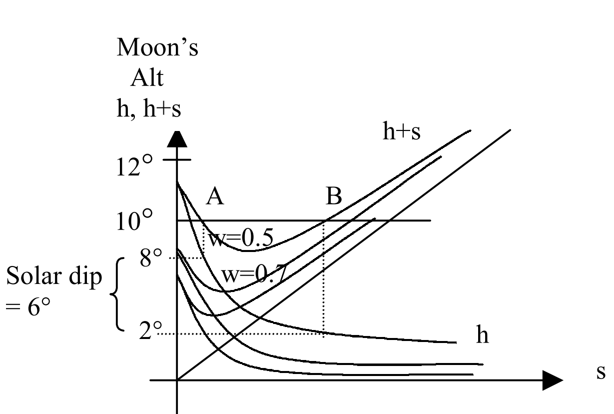


Fig. 16

Bruin's criterion is consistent internally with those observational data of Maunder's (for low latitudes).

Maunder	Bruin
$\Delta Z \geq 6°$	$(h+s) \geq 6°$ (because s=0° at local sunset)
$\Delta A_Z \approx 20°$	$a_L \approx 21.5°$ (because $\Delta A_Z = \cos^{-1} (\cos a_L / \cos\Delta Z)$ **
	⇨ $\Delta A_Z = \cos^{-1}(\cos 21.5° / \cos 6°)$
	⇨ $= 20°$)
	where $w = d \sin^2 (a_L /2)$, d = lunar diameter in km

At low attitude, the Bruin's criterion is rather accurate as it takes into account the factors mentioned above, which account for the different visibilities in different areas. Also, when plotting a histogram of fractional errors, none of the data gave a >50% error.

However, the consistency is only desirable up to low latitudes. From mid latitudes onwards, there exist great discrepancies in the results; hence, the criterion is incomplete and erroneous.

In addition, studies have shown that his criterion for visibility required a wider crescent (w>0.5 arcmin) than that for Maunder's. This requirement of crescent width was later found to be too high and reduced to >0.25 arcmin by Ilyas.

6.6 Ilyas A (Moon's Altitude, Elongation) 1984

In 1984, Ilyas plotted a curve based on observational data with the moon's altitude at sunset against the arc of light (a_L) or sun-moon elongation or the angular seperation between the sun and the moon. The crescent will be visible if the properties of the crescent lie above the curve and invisible if below the curve.

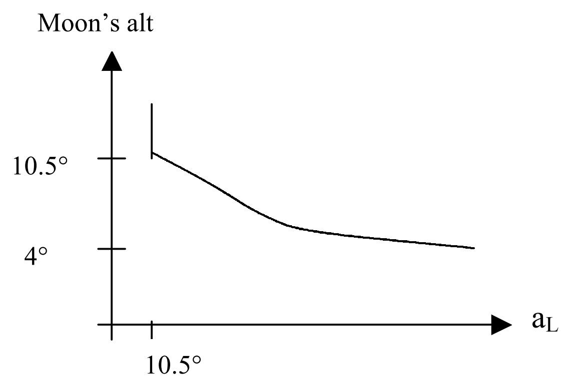


Fig. 17

Bruin	Ilyas A
From low to mid latitudes	From low to high latitudes and to very large elongation
$(h+s) \geq 6°$	$\Delta Z \geq 4°$ (extension)

6.7 Ilyas B (Lag, Latitude)

Dating back to the times of the Babylonians, the moonset lag provides one of the simplest astronomical criteria for earliest lunar visibility. However, this criterion is used mainly on lower latitudes. What happens when we talk about viewing the crescent at other latitudes? The Ilyas B criterion is then modified from the Babylonians with compensation in latitudes. The moonset lag at local sunset was calculated for each longitude for various latitudes and for 70 consecutive lunations. The results are: at lat 0° lag 41(±2) min; 30°:46 (±4) min; 40°: 49 (±9) min; 50°: 55 (±15) min).

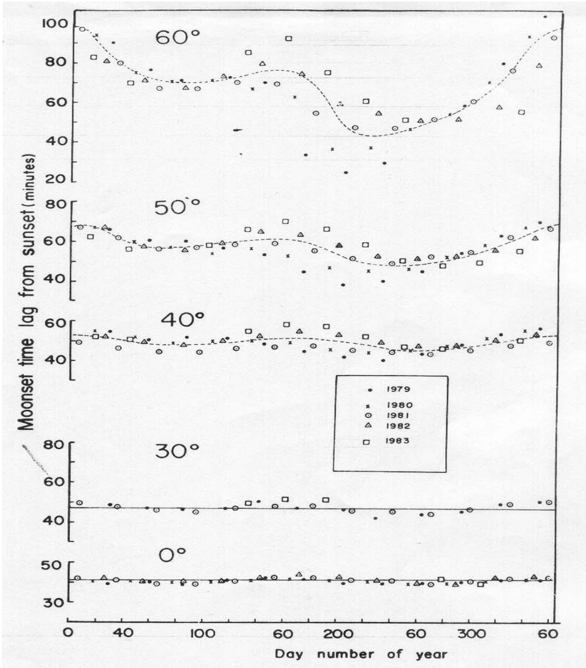


Fig. 18 [2]

We see from the resulted graph that the prediction for the earliest lunar visibility was very constant from low to mid latitudes.

However, this criterion is a simpler approximate criterion. At high latitudes (eg 60°), the data were not consistent and the graph fluctuates (data scatter). This may be due to large elongations. Note that the data have the least scatter in the spring season followed by autumn and winter, and the greatest scatter in summer. This shows that the prediction entails a large uncertainty. Here, we observe that seasonal dependence at higher latitudes is not taken into consideration for this criterion.

6.8 Ilyas C (Moon's Altitude, Relative Azimuth between moon and sun) 1988

[2] This figure is taken from [7], Ilyas, M., *A modern Guide to Astronomical Calculations of Islamic Calendar, Times and Qibla.*

In 1988, Ilyas made a slight modification of the Ilyas A criterion using a trigonometry formula**. A curve is the plotted with moon's altitude against relative azimuth. As the elongation a_L or the relative azimuth ΔA_Z is extended, the path cannot follow a because this will be overestimating; the path also cannot follow b because at high latitude, ΔZ will then become negative, ie the moon cannot be seen.

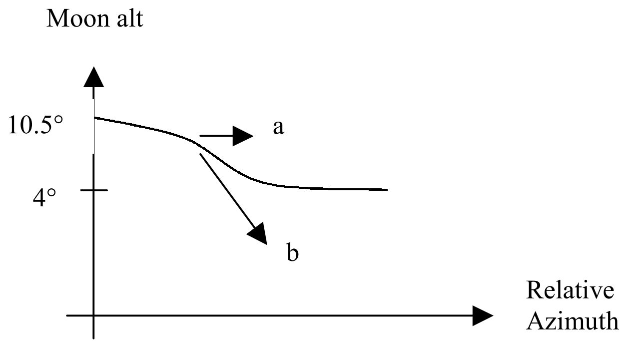


Fig. 19

Bruin	Ilyas A	Ilyas C
	ΔZ vs a_L	ΔZ vs ΔA_Z (by using $\Delta A_Z=\cos^{-1}(\cos a_L/\cos\Delta Z)$)**
	a_L =10.5°	$\Delta A_Z=\cos^{-1}(\cos 10.5° / \cos 10.5°) = 0$
(h+s) ≥6°	$\Delta Z \geq 4°$	$\Delta Z \geq 4°$

6.9 RGO (Moon's Altitude, Elongation)

RGO is the Royal Greenwich Observatory. On MoonCalc, the criterion is based on the rule that the best time and place for making the earliest sightings are when the moon is vertically above the sun at sunset so that the relative azimuth = 0° and where the altitude of the moon at sunset is 10°. From books, moon is visible when ΔZ >5° when s (solar depression) >3° and $a_L \geq 10°$. Both the criterion given in books and used by MoonCalc do agree with each other.

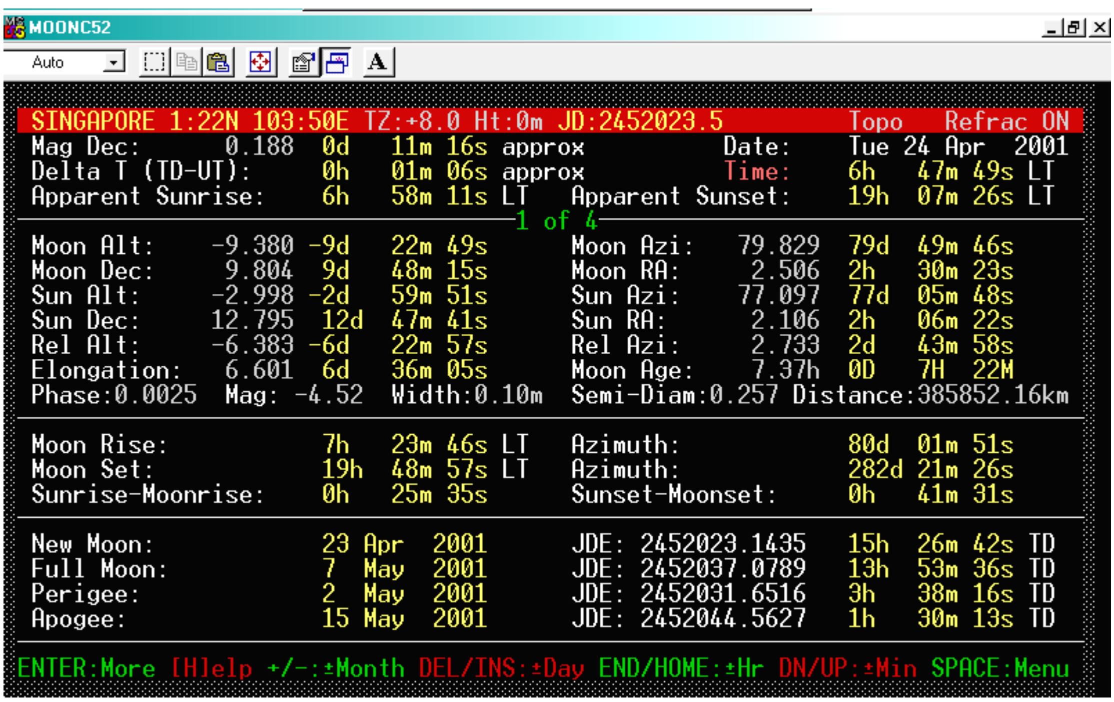


Fig. 20

We see that the sun's altitude is now >-3°, however, the moon's altitude is still below the horizon (-9.380°), and a_L =6.601° <10°, hence, the moon is not visible on 24/4/2001, according to RGO 67. Of course, when we look at the 2nd screen,

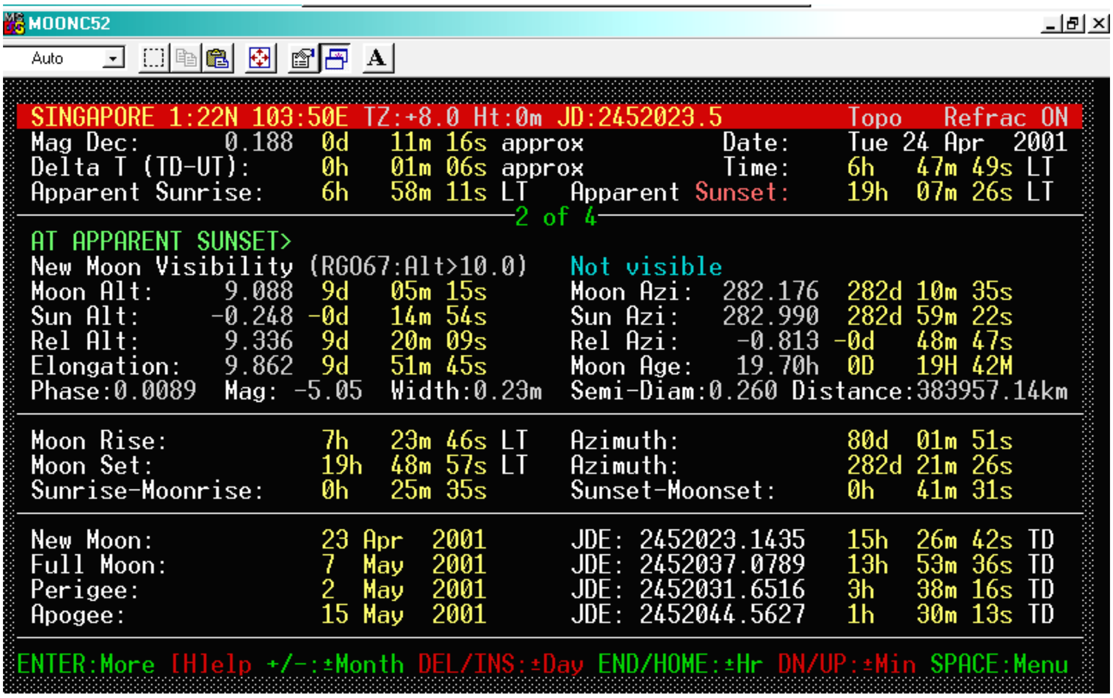


Fig. 21

we see that the moon's altitude is <10° and the relative azimuth = -0.813° ≠ 0°, hence, as shown on the screen, the moon is not visible. The moon will then be seen on the following evening.

The RGO has produced a series of information sheets which tabulates predicted first moon sightings and also has a website http://www.rog.nmm.ac.uk/index.html. At the former location of the Royal Greenwich Observatory, the prime meridian runs through the telescope of the building. The eastern hemisphere of the world, longitude east, is below it, and the western hemisphere is above it. The two hemispheres meet at 180° longitude on the opposite side of the globe.

6.10 Shaukat (Moon's Altitude, Crescent Width)

Shaukat's criterion makes use of over 900 observations, collected over a period of 150 years in different locations of the world. For every observation, he calculated all the parameters that are important for sighting. A multivariable statistics analysis is conducted to find when visibility occurs and when non-visibility occurs.
In MoonCalc: moon's altitude must be >3.4° at sunset and

(alt/12.7) + (crescent width in arcmin/1.2) > 1.

We do not know the exact criteria that led to the figures used above. Nevertheless, Khalid Shaukat is a research scientist since 1967 and has been a consultant on moon sighting for the Islamic Society of North America and had developed a website on moon sighting (http://www.moonsighting.com). His criterion has been one of those more reliable one.

6.11 Yallop (Relative Altitude, Crescent Width) 1980's

Bernard Yallop, an English astronomer, made use of 295 (non) sightings information and develop a criteria from the Indian and Bruin criteria. This criterion depends on a parameter 'q', where 'q' is derived from the relative geocentric altitude of the moon and the topocentric crescent width at 'best time', ie sunset time + (4/9) moonset lag.

However, as this criterion depends on 'best time', it is not always that accurate, because once we apply the criterion to local sunset, then the result will become more pessimistic than at 'best time'.

7 The Islamic Calendar in Saudi Arabia

7.1 The Umm al-Qurrah Calendar

Before 1420 AH, 16 April 1999, if the moon's age at sunset is ≥ 12 hours, then the **previous** day is the first day of the Islamic month, according to the old Saudi Arabian calendar criteria. (Jordanian Astronomical Society Webpage: http://www.jas.org.jo/icop.html)

For example, let 29th Dec of a certain year be 29 Sha'ban. If the moon sets before sunset and is only seen later in the night at 11 pm on 29th Dec, then at 5 pm the following day 30th Dec, the moon's age will be 18 hours (≥ 12 hours). Thus, 30th Dec will be declared as first day of Ramadan, even though conjunction does not occur and the new moon is not seen at sunset.

Starting from 1420 AH, 17 April 1999, Saudi changed their criteria. Now, Saudi starts the lunar month if the moon sets **after** the sun on the 29th day of the previous month, as seen from Mecca, otherwise, the next day is 30th of the month and the day after is the first day of the new month. Although the new criteria are much better than the old ones, it still ignores the actual sighting of the crescent. Also, at certain months they begin the month while the moon is not yet in conjunction. Setting of the moon after the sun does not always imply that the moon reached conjunction.

For example, on 7 December 1999 (29 Sha'ban), the sun will set in Mecca at 17:38 LT, and the moon will set at 17:29. So since the moon will set before the Sun, 08 December is not1 Ramadan. Consequently, 1 Ramadan will be on 9 December.

However, in most cases (*c*. 85%) the lunar crescent will still be too young to be visible with the naked eye on the first evening of the month and about once a year, the month will still commence before the new moon.

These anomalous month beginnings are caused by the fact that the sun and the moon do not set perpendicularly at the latitude of Mecca. When the moon is near its most northerly ecliptic latitude moonset can occur after sunset even before the sun and the moon have reached conjunction.

Most people think that Saudi Arabia has always adopted the actual sighting of the crescent as the basis to start Islamic month. However, previous years have shown that Saudi Arabia adopts the astronomical calculations and totally ignores the actual sighting.

7.2 The Saudi Arabian Solar Zodiac Calendar

The above calendar is officially adopted in Saudi Arabia for all religious and practical purposes. However, for some key dates Saudi Arabia also employs a solar zodiacal calendar that nearly runs parallel with the Gregorian calendar. National Day, commemorating the unification of the Kingdom of Saudi Arabia on 22 September 1932, is annually observed on the day when the Sun enters the sign of Libra, which now corresponds with either 22 or 23 September. Since 1989 the Saudi Arabian fiscal year has officially commenced on the day when the Sun reaches the longitude 11º Capricorn and runs until the day when the Sun reaches the longitude 10º Capricorn in the next year. In an average sense, these limits correspond with the Gregorian dates 1 January and 31 December – the occasional one-day shifts caused by the Gregorian leap day are disregarded.

8 The Islamic Calendar in Singapore

The Islamic Religious Council of Singapore - Majlis Ugama Islam Singapura (MUIS), releases the Islamic Calendar in Singapore. Presently, the religious ministers of Brunei, Indonesia, Malaysia and Singapore hold an unofficial meeting every year to coordinate the major Muslim holidays in Singapore with Malaysia, Brunei and Indonesia like Hari Raya. These countries decide upon the official date of the first day of every Muslim month based on the Mabims[3] criteria*:

At sunset on the 29th day of the lunar month,

(i) the moon must reach an altitude of $\geq 2°$
(ii) the sun-moon elongation must be $\geq 3°$
(iii) the age of the moon from conjunction of sunset must be ≥ 8 hrs old

However, we found out that for the Muslim calendar of Singapore, MUIS actually uses only the first criterion to determine if the new moon is visible. The other

[3] **Mabims** stands for Menteri Agama Brunei Indonesia Malaysia Singapore.

two factors only determine the degree of visibility. Later, we will illustrate this with examples.

Each country is not bound to follow these regulations due to religious or political reasons. For instance, Indonesia declares the Hari Raya Puasa date different from the rest of the 3 countries due to political reasons.

8.1 History of Islamic Calendars in Singapore

In the 50s and 60s, the Muslim calendar in Singapore was based on sightings of the moon from Sultan Shoal, the southernmost part of Singapore. In the 70s, MUIS decided to use predictions. The Muslim calendar in Singapore is based on latitude 1 20' 34" N and longitude 103 51' 08" E, where the Islamic Centre of Singapore (old MUIS building) is. If the moon was above horizon during sunset of the 29th day of the month, even if the moon is just **1°**, the new month would start. In the 80s they decided to follow a variation of the 1978 Istanbul criterion where the altitude of the moon should be more than **7°** at sunset. In the 90s, they switched to the **2°**, which is the simplification of the Mabims criteria*.

8.2 Analysis of the Islamic Calendar in Singapore based on Ilyas C, RGO and Shaukat Criteria

Based on the reliability and limitations of the above criteria, we compare Ilyas C, RGO 67 and Shaukat criteria for the Islamic Calendar in Singapore. The agreement ratio (or the yellow highlighted rows) indicates how much the predicted dates of the first day of the Muslim months agree with that of the Mabims criteria.

Calendar 1998

Month	MUIS date for the first day of the Muslim months	Mabims				Age of moon on Mabims	Agreement Ratio
			Ilyas_C	RGO 67	Shaukat		
Jan	**30/1(P)**	29/1				29.31 hr	3/12
Feb	28/2	27/2	28	28	28	17.90 hr	
Mar	29/3	28/3	29	29	29	8.00 hr	
Apr	28/4	27/4				23.43 hr	
May	27/5	26/5	27	27	27	15.59 hr	
Jun	25/6	25/6 (24/6)	25	25	25	31.38 hr	
Jul	25/7	24/7	25	25	25	21.55 hr	
Aug	23/8	22/8	23	23	23	9.15 hr	
Sep	22/9	21/9	22	22	22	18.00 hr	
Oct	22/10	21/10				24.71 hr	
Nov	20/11	20/11 (19/11)	20	20	20	30.42 hr	
Dec	20/12	19/12	20	20	20	12.34 hr	

Calendar 1999

Month	MUIS date for the first day of the Muslim months	Mabims				Age of moon on Mabims	Agreement Ratio
			Ilyas_C	RGO 67	Shaukat		
Jan	**19/1(P)**	18/1	19	19	19	19.50 hr	
Feb	18/2	17/2				28.70 hr	
Mar	19/3	18/3	19	19	19	16.47 hr	
Apr	17/4	17/4 (16/4)	17	17	17	30.78 hr	
May	17/5	16/5				23.02 hr	4/12
Jun	15/6	14/6	15	15	15	16.13 hr	
Jul	14/7	13/7	14	14	14	8.87 hr	
Aug	13/8	12/8				24.10 hr	
Sep	11/9	10/9	11	11	11	13.06 hr	
Oct	11/10	10/10				23.33 hr	
Nov	9/11	9/11 (8/11)	9	9	9	30.96 hr	
Dec	9/12	8/12	9	9	9	12.43 hr	

Calendar 2000

Month	MUIS date for the first day of the Muslim months	Mabims				Age of moon on Mabims	Agreement Ratio
			Ilyas_C	RGO 67	Shaukat		
Jan	**8/1(P)**	7/1	8	8	8	16.97 hr	
Feb	7/2	6/2		7	7	22.29 hr	
Mar	7/3	7/3 (6/3)	7	7	7	30.03 hr	
Apr	6/4	5/4	6	6	6	16.99 hr	
May	5/5	5/5 (4/5)	5	5	5	30.91 hr	3/12
Jun	4/6	3/6				22.91 hr	
Jul	3/7	2/7	3	3	3	19.91 hr	
Aug	1/8	31/7	1	1	1	8.86 hr	
	31/8	30/8				24.84 hr	
Sep	29/9	28/9	29	29	29	15.09 hr	
Oct	29/10	28/10				26.88 hr	
Nov	27/11	26/11	27	27	27	11.70 hr	
Dec	**27/12(P)**	26/12	27	27	27	17.75 hr	

The cells highlighted in blue indicate "problematic" dates, namely 25/6/**98**, 20/11/**98**, 17/4/**99**, 9/11/**99**, 7/3/**00**, 5/5/**00**. They are "problematic" because they do not follow the rule. If the moon is visible on say, 5/5/00 evening, then the official first day of that Muslim month should fall the following day, 6/5/00. In the Mabims column, the dates not in the brackets indicates the day when the all three of the Mabims criteria are fulfilled.

However, as we have mentioned above, MUIS specifies that out of the three Mabims criteria, the moon's altitude ($\geq 2°$) is the only deciding factor for visibility, while the other two will determine the degree of visibility. Let's see it more clearly on MoonCalc:

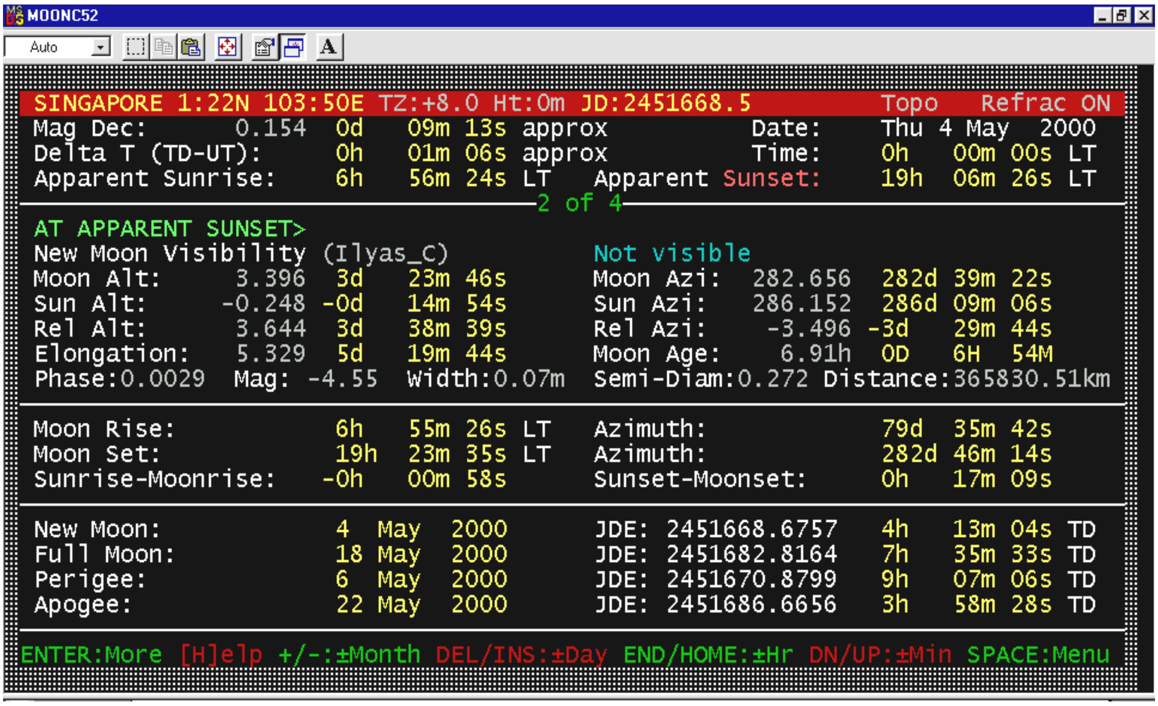


Fig. 22

<u>On 4/5/**00**</u>

Altitude of the moon is 3.396° ≥2°

Elongation is 5.329° ≥3°

Moon's Age is 6.91hr < 8hr (Does not satisfy one of Mabims criteria)

However, since the moon's altitude is already ≥ 2°, it means that the moon is visible on the evening of 4/5/**00** and not 5/5/**00**.

Calendar 2001

Month	MUIS date for the first day of the Muslim months	Mabims				Age of moon on Mabims	Agreement Ratio
			Ilyas_C	RGO 67	Shaukat		
Jan	26/1	25/1	26	26	26	22.20 hr	
Feb	25/2	24/2				26.99 hr	
Mar	26/3	25/3	26	26	26	9.88 hr	
Apr	25/4	24/4		25	25	19.70 hr	
May	24/5	23/5	24	24	24	8.35 hr	4/12
Jun	23/6	22/6				23.25 hr	
Jul	22/7	21/7	22	22	22	15.54 hr	
Aug	20/8	19/8	20	20	20	8.30 hr	
Sep	19/9	18/9				24.58 hr	
Oct	18/10	17/10	18	18	18	15.49 hr	
Nov	17/11	16/11				28.19 hr	
Dec	**16/12(P)**	15/12	16	16	16	14.23 hr	

Calendar 2002

<u>**Age of moon on**</u>

Month	MUIS date for the first day of the Muslim months	Mabims				Mabims	Agreement Ratio
			Ilyas_C	RGO 67	Shaukat		
Jan		14/1		15	15	21.77 hr	
Feb	14/2	13/2				27.67 hr	
Mar		14/3	15	15	15	9.24 hr	
Apr		13/4	14	14	14	15.81 hr	
May		13/5		14	14	24.35 hr	4/12
Jun		11/6	12	12	12	11.40 hr	
Jul		11/7				24.83 hr	
Aug		9/8	10	10	10	16.00 hr	
Sep		8/9 (7/9)	8	8	8	31.94 hr	
Oct		7/10				23.63 hr	
Nov		5/11	6	6	6	14.26 hr	
Dec	**6/12(P)**	5/12				27.37 hr	

Calendar 2003

Month	MUIS date for the first day of the Muslim months	Mabims				Age of moon on Mabims	Agreement Ratio
			Ilyas_C	RGO 67	Shaukat		
Jan		3/1	4	4	4	14.79 hr	
Feb	3/2	2/2				24.53 hr	
Mar		3/3	4	4	4	8.74 hr	
Apr		2/4	3	3	3	15.89 hr	
May		2/5	3	3	3	22.87 hr	5/12
Jun		1/6				30.81 hr	
Jul		30/6	1	1	1	16.60 hr	
		30/7				28.40 hr	
Aug		28/8		29	29	17.74 hr	
Sep		27/9 (26/9)	27	27	27	31.83 hr	
Oct		26/10				22.01 hr	
Nov	**25/11(P)**	24/11	25	25	25	11.90 hr	
Dec		24/12				25.37 hr	

Calendar 2004

Month	MUIS date for the first day of the Muslim months	Mabims				Age of moon on Mabims	Agreement Ratio
			Ilyas_C	RGO 67	Shaukat		
Jan	23/1	22/1	23	23	23	14.21 hr	
Feb		21/2				26.05 hr	
Mar		21/3	22	22	22	12.56 hr	
Apr		20/4	21	21	21	21.78 hr	
May		20/5				30.25 hr	3/12
Jun		18/6	19	19	19	14.75 hr	

Jul		18/7				23.88 hr	
Aug		16/8	17	17	17	9.83 hr	
Sep		15/9		16	16	20.57 hr	
Oct		14/10	15	15	15	8.08 hr	
Nov	**14/11(P)**	13/11		14	14	20.39 hr	
Dec		12/11	13	13	13	9.52 hr	

Calendar 2005

Month	MUIS date for the first day of the Muslim months	Mabims				Age of moon on Mabims	Agreement Ratio
			Ilyas_C	RGO 67	Shaukat		
Jan	12/1	11/1				23.19 hr	
Feb		9/2	10	10	10	12.88 hr	
Mar		11/3				26.12 hr	
Apr		9/4	10	10	10	14.64 hr	
May		9/5				26.35 hr	3/12
Jun		7/6	8	8	8	13.24 hr	
Jul		7/7		8	8	23.22 hr	
Aug		5/8	6	6	6	8.18 hr	
Sep		4/9	5	5	5	16.37 hr	
Oct		4/10		5	5	24.47 hr	
Nov	**4/11(P)**	2/11	3	3	3	9.43 hr	
Dec		2/12	3	3	3	19.91 hr	

In 2005, we observe that according to Mabims criteria on MoonCalc, the crescent can be seen on 2/11/**05**, yet, the official Hari Raya Puasa falls on 4/11/**05**. The question is, why isn't Hari Raya Puasa on 3/11/**05**, since the crescent is visible on 2/11/**05**? This is because, the values on MoonCalc are slightly different from that of the programme used by MUIS. Below is the data that you would see on MoonCalc for 4/11/**05**:

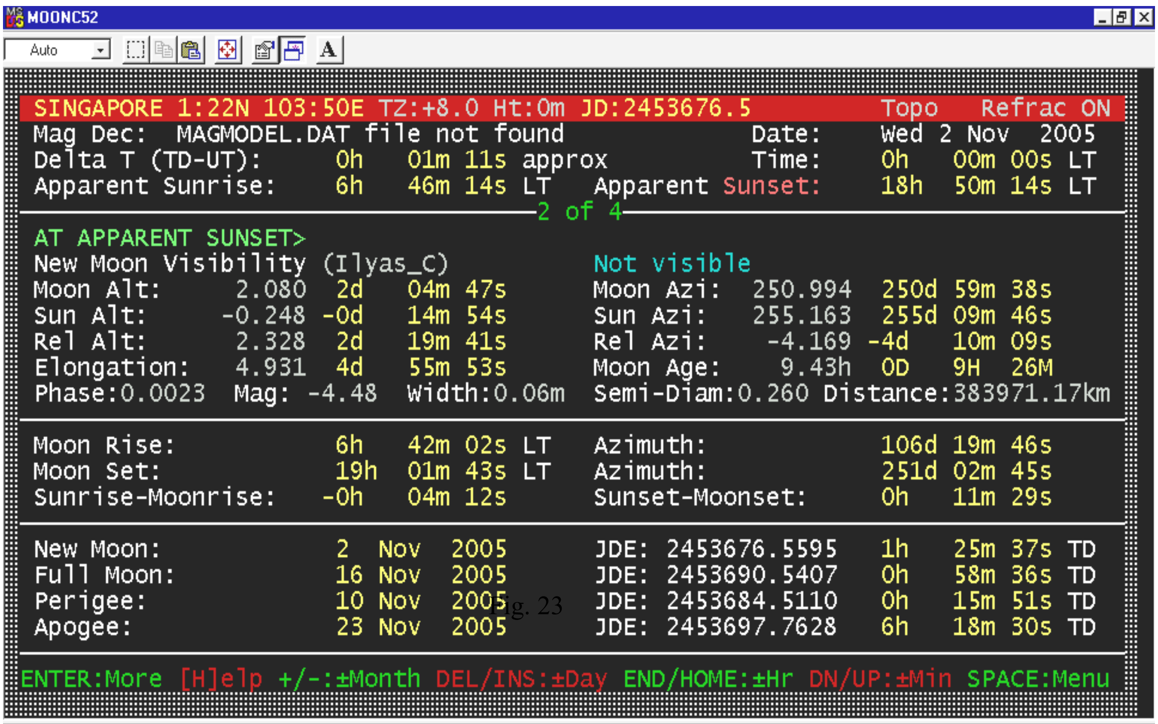


Fig. 23

Altitude of the moon is 2.080° ≥ 2°
Elongation is 4.931° ≥ 3°
Moon's Age is 9.43h ≥ 8h
All three of the Mabims criteria (using MoonCalc) are satisfied. However, the programme that MUIS uses shows an altitude of the moon to be < 2° (i.e. the moon is not above the horizon at local sunset and hence, cannot be seen). The moon will then be seen only on the following evening and 4/11/**05** will be Hari Raya Puasa.

Extending our analysis to the agreement ratio between the 3 criteria and Mabims, we realize that they agree for at most 5 (out of 12) months per year. While Mabims criteria is decided upon by the 4 countries, we do no know the basis for these results.

9 Conclusion

We have seen how Hari Raya Puasa and Hari Raya Haji (or more precisely, the first day of the Muslim month) can be determined, be it by physical moon-sighting or prediction criteria. Yet, the problem of predicting when the new moon will become visible is indeed both astronomical and physical. The visibility of the thin lunar crescent is of cultural and religious importance, since the lunar phase cycle is the basis for the Islamic calendars of the Muslim communities. Therefore, the determining of the first day of the month based on physical moon sighting always carries the essence of the Muslim religion. However, physical moon sighting has its own errors and limitations too; and for countries that do not have clear skies to observe the moon like Singapore, calendar planning is important for festive celebrations. Thus, criteria which control the phenomenon are being studied with great effort. With so much discussion on the different prediction criteria, we see that none of them is 100% accurate. Therefore, these criteria are being looked into year by year to obtain greater accuracy. One may question the reliability of astronomical software being used today. But MoonCalc is considered to be one of the more reliable ones. You may ask if there is the possibility of having a worldwide Islamic calendar for all the countries to celebrate the Muslim festivals together. Nevertheless, some countries use a slightly different set of criteria from others because of religious reasons, while others do not agree upon the same date of Muslim festivals due to political reasons. What is hoped to achieve in the future development of the Islamic Calendar then, is the improvement of the accuracy of the prediction criteria.

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